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SOLUTION BY THE PROPOSER.

I shall first prove some lemmas.

1) If x, y, and u are integers such that $x^2 + y^2 = u^2$, then either x or y is divisible by 3. This follows from the fact that any perfect square is congruent, modulo 3, to either 0 or 1.

2) If x, y, and u are integers such that $x^2 + y^2 = u^2$, then either x or y is divisible by 4; for any perfect square is congruent, modulo 16, to 0, 1, 4, or 9.

3) If x, y, and u are integers such that $x^2 + y^2 = u^2$, then at least one of the numbers x, y, u is divisible by 5; for any perfect square is congruent, modulo 5, to 0, 1, or 4.

Hence, from the hypotheses, at least two of the numbers x, y, z are divisible by 3. If the notation be so chosen that these are x and y, the first equation shows that u also is divisible by 3. We can then divide both terms of this equation by 9; it then follows that either $x/3$ or $y/3$ is divisible by 3. Thus the product $xyzuvw$ is divisible by 3^4. In the same way it can be proved that the product is divisible by 4^4.

By lemma 3, at least one of the numbers x, y, u is divisible by 5. Similarly, at least one of each of the sets x, z, v and y, z, w is divisible by 5. Thus at least two of the numbers x, y, z, u, v, w are divisible by 5.

Therefore the product $xyzuvw$ is divisible by $3^4 \cdot 4^4 \cdot 5^2$, or 518400.

Also solved by H. C. FEEMSTER and J. L. RILEY.

2661. Proposed by ARTEMAS MARTIN, Washington, D. C.

Find a parallelepipedon whose edges, and the diagonals of its faces, are all rational whole numbers.

SOLUTION BY THE PROPOSER.

Denote the edges by x, y, and z; then

$$x^2 + y^2 = \square, \quad x^2 + z^2 = \square, \quad y^2 + z^2 = \square. \tag{1, 2, 3}$$

Assume $x = 2pq$, $y = p^2 - q^2$; then by substitution,

$$x^2 + y^2 = (2pq)^2 + (p^2 - q^2)^2 = (p^2 + q^2)^2,$$

a square;

$$x^2 + z^2 = (2pq)^2 + z^2, \quad y^2 + z^2 = (p^2 - q^2)^2 + z^2, \tag{4, 5}$$

which must be made squares.

Put

$$(2pq)^2 + z^2 = \left(z - \frac{2pqr}{s}\right)^2,$$

which gives

$$z = \frac{pq(r^2 - s^2)}{rs}.$$

Substituting in (5) the value of z found above,

$$(p^2 - q^2)^2 + \frac{p^2q^2(r^2 - s^2)^2}{r^2s^2} = \square,$$

or

$$r^2s^2(p^2 - q^2)^2 + p^2q^2(r^2 - s^2)^2 = \square. \tag{6}$$

Expanding (6) it may be written

$$(p^2r^2 + q^2s^2)(q^2r^2 + p^2s^2) - 4p^2q^2r^2s^2 = \square,$$

which will be the case when

$$q^2r^2 + p^2s^2 = 4q^2s^2. \tag{7}$$

By transposition, (7) becomes

$$q^2r^2 = 4q^2s^2 - p^2s^2 = s^2(4q^2 - p^2) = s^2\left(\frac{mp}{n} - 2q\right)^2,$$

say; and we get

$$\frac{p}{q} = \frac{4mn}{m^2 + n^2}, \quad \frac{r}{s} = \frac{2(m^2 - n^2)}{m^2 + n^2};$$

$$x = 2pq = 8mn(m^2 + n^2), \qquad y = p^2 - q^2 = (4mn)^2 - (m^2 + n^2)^2,$$

$$z = \frac{pq(r^2 - s^2)}{rs} = \frac{2mn[4(m^2 - n^2)^2 - (m^2 + n^2)^2]}{m^2 - n^2}.$$

Reducing the foregoing values of x, y, z to a common deominator $m^2 - n^2$ and then discarding it, we finally have

$$x = 8mn(m^2 - n^2)(m^2 + n^2),$$

$$y = (m^2 - n^2)[(4mn)^2 - (m^2 + n^2)^2],$$

$$z = 2mn[4(m^2 - n^2)^2 - (m^2 + n^2)^2].$$

Take $m = 2$, $n = 1$ and we get $x = 240$, $y = 117$, $z = 44$, the smallest values known. The diagonals of the faces are 267, 244, 125.

Also solved by C. B. HALDEMAN and L. E. LUNN.

2664. Proposed by J. W. NICHOLSON, Baton Rouge, La.

Find the sum of the series, $\frac{1}{3} - \frac{2}{15} + \frac{3}{35} - \cdots + (-1)^{n+1} \frac{n}{(2n-1)(2n+1)}$.

SOLUTION BY S. W. REAVES, University of Oklahoma.

The exponent of -1 in the general term should obviously be $n + 1$. Making this correction, the general term may be written,

$$(-1)^{n+1} \frac{n}{(2n-1)(2n+1)} = \tfrac{1}{2}(-1)^{n+1}\left(\frac{n}{2n-1} - \frac{n}{2n+1}\right).$$

Let S denote the required sum. Then

$$S = \frac{1}{2}\left[\sum_{i=1}^{n} (-1)^{i+1} \frac{i}{2i-1} + \sum_{i=1}^{n} (-1)^{i+2} \frac{i}{2i+1}\right]$$

$$= \frac{1}{2}\left[\left(1 - \tfrac{2}{3} + \tfrac{3}{5} - \cdots + (-1)^{n+1} \frac{n}{2n-1}\right) + \left(\tfrac{1}{3} + \tfrac{2}{5} - \cdots + (-1)^{n+2} \frac{n}{2n+1}\right)\right]$$

$$= \frac{1}{2}\left[1 - 1 + 1 - 1 + \cdots + (-1)^{n+2} \frac{n}{2n+1}\right].$$

If n be even, $S = \dfrac{n}{4n+2} = \tfrac{1}{4}$ when n is infinite.

If n be odd, $S = \dfrac{n+1}{4n+2} = \tfrac{1}{4}$ when n is infinite.

Also solved by BANCROFT H. BROWN, E. H. WORTHINGTON, E. B. ESCOTT, GEORGE F. WILDER, PAUL CAPRON, HORACE OLSON, ELIJAH SWIFT, and ARNOLD DRESDEN.

2665. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A telegraph wire, which weighs 1/10 lb. per yard, is stretched between poles on a level ground so that the greatest dip of the wire is 3 feet. Find, approximately, the distance between the poles when the tension at the lowest point of the wire is 140 pounds.

SOLUTION BY G. PAASWELL, New York City.

MacMahon's *Hyperbolic Functions*, pages 47 to 51 discusses the two cases here, (*a*) when the wire is taken as inextensible, and (*b*) as extensible. Under (*a*) $y/c = \sec \varphi$; $x/c = gd^{-1} \varphi = \log_e \tan (\pi/4 + \varphi/2)$. The origin is the point midway between the poles and a distance c below the lowest point of the wire. Use the yard as the unit of length. $c = H/w = 140/0.1 = 1400$. $y = 1 + c = 1401$. Whence, $\varphi = 2° 9' 54''$. Thence, $x = 52.913$ and since $L = 2x$, the re-